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AXIAL CONDUCTION IN LAMINAR PIPE FLOWS WITH NONLINEAR WALL HEAT FLUXES

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Abstract—A numerical analysis to determine the heat-transfer parameters of a fluid flow rejecting heat to the surrounding medium by convection and radiation is developed. The influence of axial conduction is included and the velocity profile is taken as nonuniform in the transverse direction. Use of a transformation eliminates the required boundary conditions at infinity. Approximate numerical techniques are employed to solve the nonlinear conjugate problem. As Péclet number increases, the temperature fields simplify to those where axial conduction is excluded. The computed results indicate that the effects of axial conduction are strongly altered by the parameters responsible for the convection and radiation. Bulk fluid temperatures, wall heat fluxes and Nusselt numbers are plotted against Graetz numbers. Critical Péclet numbers for a variety of cooling conditions are presented using the bulk fluid temperature as a reference.

NOMENCLATURE

A^*, B^*, coefficients in equation (13);
Bi, Biot number, hR/k;
D, tube diameter [m];
G_z, Graetz number, z/RPe;
h, external convection coefficient [W/m^2°C];
h', internal convection coefficient [W/m^2°C];
k, fluid thermal conductivity [W/m°C];
K, transformation constant in equation (12);
L, number of axial increments;
M, number of radial increments;
N, number of equations, $M \times L$;
Nu, Nusselt number, $h'D/k$;
Pe, Péclet number, sDv_m/k;
q, heat flux;
Q, dimensionless heat flux, qR/kT_e;
r, radial distance [m];
r', normalized radial distance, r/R;
R, tube radius [m];
s, volumetric heat capacity [kJ/m^3°C];
Sk, Stark number, $\varepsilon\sigma RT_e^3/k$;
T, absolute temperature when radiation is present [°K];
U, normalized temperature, T/T_e;
v, velocity [m/s];
v_m, mean velocity [m/s];
z, axial distance [m].

Greek symbols
ε, tube emissivity;
ε_{local}, convergence criterion;
η, normalized axial distance in equation (12);
σ, Stefan–Boltzmann constant [W/m^2 K^4].

Subscripts
a, convection sink;
b bulk;
c, critical;
e, entrance;
i, value at the axial position;
j, value at the radial position;
i,j, value at the node;
L, local;
s, effective radiation sink;
w, wall;
o, origin.

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INTRODUCTION

THE CONTRIBUTION of axial heat conduction plays a significant role in the analysis and design of heat-transfer equipment using low Péclet number fluids. Although there is an extensive literature dealing with this particular problem, its mathematical representation had not been established with certainty until recently. The apparently conflicting trends of the initial studies are summarized and examined in a publication by Hennecke [1] in 1968. He stated that the problem cannot be formulated by simply adding the axial conduction term to the energy equation while using a semi-infinite duct with a uniform inlet temperature. This dilemma is attributed to the fact that the unrealistic boundary condition at the origin $z = 0$ negates the conduction of heat upstream of the entrance point. Therefore, since the temperature at $z = 0$ is not known a priori, it is necessary to employ the geometry of an infinite duct where the temperature is constant at $z = -\infty$. Thus, a complete solution of the properly posed problem requires additional consideration of the energy equation. This modification adds drastic computational difficulties to the solution of the problem.

Utilizing a finite difference procedure Hennecke [1] solved the governing energy equation for a Poiseuillian flow through circular tubes in the infinite region $-\infty \leqslant z \leqslant \infty$. His numerical results are applicable for two different sets of thermal boundary conditions, i.e. the uniform wall temperature and the uniform heat flux. Both conditions show the customary discontinuities at the origin. The distorted temperature profiles are illustrated in a group of curves assessing the importance of the axial conduction phenomenon.

Subsequent publications related to this problem have been concerned basically with different mathematical treatments involving variations of the same boundary conditions used in [1]. Jones [2–4] presented a theoretical solution based on the application of a two-sided Laplace transform and obtained both the upstream and downstream temperature fields. Two separate cases dealing with boundary conditions of the first and second kind were investigated. Deavours [5] found an exact solution for the temperature profile of a fluid flow between parallel plates. One semi-infinite portion of the plate walls is maintained at a fixed temperature while the other is maintained at a different fixed temperature.

A mathematical scheme for solving the convection problem with a step change in wall heat flux at $z = 0$ was devised by Hsu [6, 7]. The scheme consists in matching the temperature distributions calculated for the regions $z \leqslant 0$ and $z \geqslant 0$ respectively. To accomplish this process the Gramm–Schmidt orthonormalization technique was employed. For the case of flow inside a circular pipe the scheme yields Nusselt numbers that agree with those reported in [1]. An analytical procedure equivalent to that of Hsu was presented by Davis [8] for the situation of fixed heat flux at the walls. The solution was expressed in terms of the confluent hypergeometric function. Pearson and Wolf [9] examined the situation of a three zone channel formed by two infinite planes. The walls of the inlet and exit zones are adiabatic but the central zone has an axially-dependent heat flux. They obtained numerical approximations developed through the application of finite element methods. Smith *et al.* [10] extended the system considered in [6] and accounted for internal energy sources. The wall heat flux varies around the circumference but is unaltered in the axial direction except for a single discontinuity. A series solution exploits the orthogonality relationship among solution components.

The following investigators employed a different set of thermal boundary conditions. Michelsen and Villadsen [11] analyzed the problem of heat transfer for Poiseuille flow under the assumption that the tube wall is kept insulated upstream of the origin and at a constant temperature downstream. The partial differential equation was solved by a method based on a combination of orthogonal collocation and matrix diagonalization. The model of plug flow was used by Jerri and Davis [12] to show that the problem given in [11] can be solved by applying the generalized sampling theorem. As a result, it provides a relation for the coefficients of the temperature fields in the two domains. Sorensen and Stewart [13] reformulated the situation presented by [9], but in this case, the central segment of a circular duct was maintained at a constant temperature. Approximate solutions for the temperature profile were obtained through the use of a collocation procedure. A numerical solution that avoids the boundary condition at infinity was developed by Verhoff and Fisher [14]. The inverse-tangent transformation converts the axial boundary conditions into coordinates located at finite distances from the origin. Constant-wall-temperature and insulation constant-wall-temperature cases were investigated.

All of these investigations [1–14] have contributed to the qualitative description of the Graetz problem accounting for longitudinal conduction. It is interesting to note that linear thermal boundary conditions imposed on the walls have been limited to situations involving constant temperature and constant or position-dependent heat fluxes. Examination of the literature shows that there are fields of application where these simple thermal boundary conditions do not apply [15]. Therefore, it suggests a need for a better understanding of the heat transfer encountered when more general conditions are essential. It is important to mention that the case of constant temperature is valid only when the tube flow is exposed to a high intensity of external forced convection. However, for intermediate external convection, heat transfer calculations made previously are not related to low Péclet number fluids. Moreover, increases in operating temperatures have reached the point where the heat flux levels are not adequately described by convection alone. Under these circumstances, convective–radiative boundary conditions need to be used. Here again, heat-transport calculations have been restricted to flows where the axial conduction is absent.

The present work solves the laminar convection problem including the effects of axial fluid conduction. General thermal boundary conditions accounting for the simultaneous role of convection and radiation downstream of the origin are employed. The upstream part of the origin is maintained insulated. Solutions are obtained by solving numerically the partial differential equation governing the resulting nonlinear conjugate problem. The implicit formulation gives rise to a system of nonlinear algebraic equations. This system is solved by the

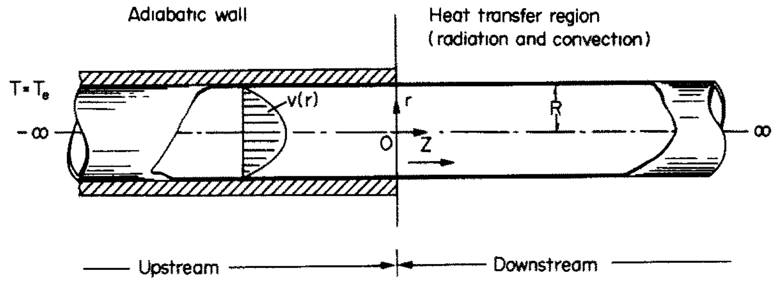

FIG. 1. Physical problem.

combined use of the Gauss–Seidel and Newton–Raphson iterative techniques. Heat-transfer parameters evaluated from the present study are compared with limiting results in order to test the generality of the model. In the presentation of results, the influence of thermal radiation and external convection on the axial conduction are examined in detail.

PHYSICAL FORMULATION

The analysis is based on a fully developed laminar flow of a viscous fluid with axial conduction in a circular tube. The upstream part of the tube from $z = -\infty$ to $z = 0$ is insulated while its downstream part from $z = 0$ to $z = \infty$ is allowed to exchange heat with the surroundings. This exchange occurs by a combined mechanism of radiation and convection. The incompressible fluid with constant properties enters the tube at $z = -\infty$ having a uniform temperature T_e. The heat flux at the wall surface is zero for $z<0$, and at the origin there is a step change in heat flux becoming nonlinear for $z \geqslant 0$. Referring to the coordinate system shown in Fig. 1, the temperature distribution for $-\infty<z<\infty$ is determined by solving the energy conservation equation for the fluid

$$\frac{s}{k} v \frac{\partial T}{\partial z} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \quad (1)$$

where the Hagen–Poiseuille velocity profile is expressed by

$$v/2v_m = 1-(r/R)^2. \quad (2)$$

The appropriate boundary conditions for the conjugate problem are

$$T = T_e \quad z = -\infty, 0 \leqslant r \leqslant R \quad (3)$$

$$\frac{\partial T}{\partial r} = 0 \quad z<0, r = R \quad (4)$$

$$\frac{\partial T}{\partial r} = 0 \quad -\infty \leqslant z \leqslant \infty, r = 0 \quad (5)$$

$$-k\frac{\partial T}{\partial r} = h(T-T_a)+\varepsilon\sigma(T^4-T_s^4) \quad z \geqslant 0, r = R \quad (6)$$

$$\frac{\partial T}{\partial z} = 0 \quad z = \infty, 0 \leqslant r \leqslant R. \quad (7)$$

It is known that the uniform temperature boundary condition at $z = 0$ is physically unrealistic for situations involving axial conduction. According to investigations by Hennecke [1] and Michelsen and Villadsen [11], the heat conducted upstream of $z = 0$ is significant when $Pe = 50$ and 20 for constant wall temperature and constant wall heat flux respectively. Therefore, for these situations the phenomenon of axial heat conduction is dictated by a single critical value of the deciding parameter Pe. However, a completely different state of affairs occurs when the upstream conduction is affected by the simultaneous presence of convection and radiation in the downstream region. For this general case, the critical values of the Péclet number are strongly affected by the combination of external convection, radiation and the temperatures of the convective and radiative sinks respectively. Equation (6), written for a general case of coupled heat flow, reduces to the particular cases of either constant wall temperature or constant wall heat flux in the downstream region [1–8, 11–14]. Since in general the sink temperatures are different, equation (7) permits the calculation of the corresponding equilibrium temperature at $z = \infty$. The equilibrium temperature at this location is independent of the hypothesis of axial conduction.

The temperature solutions of equations (1)–(7) permit the computation of certain thermal quantities of practical interest. First, the bulk fluid temperature is defined as

$$T_b = \frac{\int_0^R Tvr\,dr}{\int_0^R vr\,dr} \quad -\infty \leqslant z \leqslant \infty. \quad (8)$$

Meanwhile, the wall temperature T_w introduced in equation (6) permits the direct evaluation of the wall heat flow

$$q_w = h(T_w-T_a)+\varepsilon\sigma(T_w^4-T_s^4) \quad z \geqslant 0. \quad (9)$$

Consequently, the expression for the Nusselt number may be written in terms of q_w as follows

$$Nu = \frac{h'D}{k} = \frac{D}{k}\frac{q_w}{(T_b-T_w)} \quad z \geqslant 0 \quad (10)$$

where h' denotes the internal convection coefficient.

By introducing the normalized variables $r' = r/R$, $U = T/T_e$ and the Péclet number $Pe = sDv_m/k$ the energy equation (1) becomes

$$Pe(1-r'^2)R\frac{\partial U}{\partial z} = \frac{\partial^2 U}{\partial r'^2} + \frac{1}{r'}\frac{\partial U}{\partial r'} + R^2\frac{\partial^2 U}{\partial z^2}. \quad (11)$$

To complete the set of dimensionless variables in equation (11), the axial position z will be transformed according to the relation employed in [14]:

$$\eta = \frac{1}{\pi}\tan^{-1}\frac{z}{KR} \quad (12)$$

where K is a transformation constant. This manipulation has the advantage that not only the new axial position is dimensionless; but more important, the boundary conditions at $z = \pm\infty$ are now converted to finite locations at $\eta = \pm 0.5$. The role of this transformation is especially significant when numerical methods are attempted. Using the chain rule for derivatives and after rearranging terms, equation (11) may be rewritten in the following form

$$A^*\frac{\partial U}{\partial \eta} = \frac{\partial^2 U}{\partial r'^2} + \frac{1}{r'}\frac{\partial U}{\partial r'} + B^*\frac{\partial^2 U}{\partial \eta^2} \quad (13)$$

where

$$A^* = \frac{\cos^2(\pi\eta)}{K\pi}\left[Pe(1-r'^2) + \frac{\sin(2\pi\eta)}{K}\right]$$

and

$$B^* = \frac{\cos^4(\pi\eta)}{(K\pi)^2}.$$

Likewise, the boundary conditions are expressed as

$$U = 1 \quad \eta = -0.5, 0 \leqslant r' \leqslant 1 \quad (14)$$

$$\frac{\partial U}{\partial r'} = 0 \quad -0.5 \leqslant \eta < 0, r' = 1 \quad (15)$$

$$\frac{\partial U}{\partial r'} = 0 \quad -0.5 \leqslant \eta \leqslant 0.5, r' \doteq 0 \quad (16)$$

$$-\frac{\partial U}{\partial r'} = Bi(U-U_a) + Sk(U^4 - U_s^4)$$

$$0 \leqslant \eta \leqslant 0.5, r' = 1 \quad (17)$$

$$\frac{\partial U}{\partial \eta} = 0 \quad \eta = 0.5, 0 \leqslant r' \leqslant 1. \quad (18)$$

The existence of the axial conduction term and the presence of variable coefficients in equation (13), combined with the nonlinear boundary condition of equation (17) imply that exact mathematical techniques are not amenable. Therefore, the temperature field will be computed via the calculus of finite differences.

In terms of the adopted normalized variables, the expression for the fluid bulk temperature becomes

$$U_b = 4\int_0^{r'} r'(1-r'^2)U\,\mathrm{d}r' \quad -0.5 \leqslant \eta \leqslant 0.5. \quad (19)$$

This equation is integrated by numerical procedures using Simpson's rule. Next, the wall heat flow can be obtained from the relation

$$\frac{Q_w}{Q_{-\infty}} = \frac{Bi(U_w - U_a) + Sk(U_w^4 - U_s^4)}{Bi(1-U_a) + Sk(1-U_s^4)} \quad \eta \geqslant 0. \quad (20)$$

Finally, the Nusselt number is rewritten as follows

$$Nu_L = \frac{2[Bi(U_w - U_a) + Sk(U_w^4 - U_s^4)]}{(U_b - U_w)} \quad \eta \geqslant 0. \quad (21)$$

COMPUTATIONAL METHOD

The finite-difference formulation of equation (13) along with the associated boundary conditions, equations (14)–(18) were solved numerically on a digital computer. It is known that the energy equation accounting for axial conduction effects is classified as elliptic. Hence, its representation in difference form is carried out by means of the implicit technique. Consequently, a system of nonlinear algebraic equations is generated due to the convection-coupled radiation boundary condition in the downstream region.

The tube was divided into rectangular cells, and the radial and axial divisions being designated by $\Delta r'$ and $\Delta\eta$ respectively. The subindices i and j are assigned to the radial and axial directions respectively. Using this notation, the central difference analog is utilized for the conversion of all derivatives appearing in equations (13)–(18).

Therefore, a nonlinear system of equations consisting of $N = M \times L$ equations is generated. The numerical approach employed for its solution is based on the Gauss–Seidel method and the Newton–Raphson method. The accuracy of the iteration procedure was influenced by three factors: the convergence criterion $\varepsilon_{\text{local}}$, the grid size $M \times L$ and the transformation constant K given by equation (12). Using limiting cases for the physical problem, it was possible to arrive at the numerical values of $\varepsilon_{\text{local}}$, $M \times L$ and K that assure reliable results. The cases tested correspond to combinations of low axial conduction ($Pe = 30$) or high axial conduction ($Pe = 1$) with high cooling levels ($Bi = 20$ and $Sk = 10$) or low cooling levels ($Bi = 1$ and $Sk = 1$). The selected value for $\varepsilon_{\text{local}}$ corresponds to a small change in each nodal temperature between two consecutive iterations and is given by $\varepsilon_{\text{local}} \leqslant 0.001\%$.

For $Pe = 1$ and $Bi = 20$, $Sk = 10$ the calculated results do not exhibit appreciable discrepancies using grid sizes of 10×40, 20×40 and 10×60 respectively. On the other hand, for $Pe = 30$ with the same cooling conditions, good convergence is achieved for a grid of 10×60 only. Consequently, a grid with 10 radial intervals and 60 axial intervals along with a local convergence criterion $\varepsilon_{\text{local}} = 0.001\%$ seemed to yield accurate results for computation purposes. These quantities are used to determine an appropriate value for the transformation constant K. When $K = 0.2$ and 1.0, Fig. 2 reveals that minor variations of wall temperatures take place in the neighborhood of $G_z = 0$. It is also observed that the variations tend to disappear at points distant from the origin.

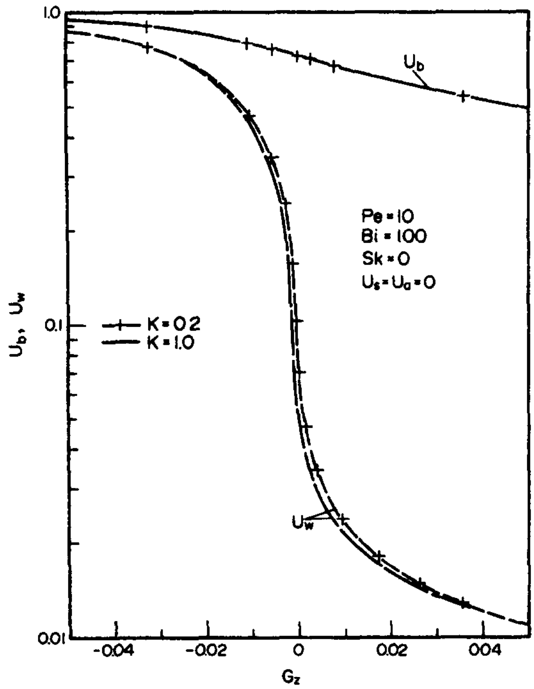

FIG. 2. Influence of K near the origin.

Moreover, it was found that small values of K furnish better results in the vicinity of $G_z = 0$. At distant locations from the origin, better results are obtained for large values of K. Therefore, an intermediate value of K, say $K = 0.4$ was attempted. Additionally, Fig. 2 shows that the bulk fluid temperature is insensible to changes in K.

To establish the validity of the numerical approach employing $K = 0.4$, a comparison using the results of Michelsen and Villadsen [11] are presented in Fig. 3. Small differences observed in the values of U_b for $Pe = 10$ and ∞ are attributed to the fact that the results of [11] are valid for a constant wall temperature ($U_w = 0$). This boundary condition constitutes a limiting approximation to the present case $Bi = 100$, $Sk = 0$. As a result, Fig. 3 also shows that U_w is not identically zero. Consequently, $K = 0.4$ appears to be a reasonable choice and was adopted for all the calculations.

RESULTS

Some insights into the physical phenomenon of axial conduction can be gained by comparing the results with those where axial conduction is excluded. Numerical calculations for U_b, $Q_w/Q_{-\infty}$ and Nu_L corresponding to the case of $Pe = \infty$ are taken from Campo and Auguste [16]. This reference involves viscous heating and does not include axial conduction, but otherwise makes the same assumptions as in the present analysis.

Computed results are presented graphically in Figs. 4–13. For simplicity, sink temperatures are maintained at intermediate values of $U_a = 0.4$ and $U_s = 0.4$, when applicable.

The first set of Figs. 4–9 have common cooling conditions: $Bi = 2.5$ and $Sk = 1$. The fluid bulk temperature U_b as a function of the axial distance G_z, having Pe as a parameter is shown in Fig. 4. Temperature solutions approach those presented in [16] as Pe increases. This expected behavior implies that for large Pe, axial conduction does not contribute significantly to the heat transport mechanism. The inset of Fig. 4 tabulates the bulk temperatures at the origin U_{b_0} in terms of Pe. It is seen that $U_{b_0} = 0.5521$ when $Pe = 1$ which is about half of the entrance temperature at $G_z = -\infty$. Figure 5 illustrates the temperature distributions at $G_z = 0$ that give rise to the previous tabulation. The effects of axial conduction altering the uniform temperature pattern at the origin is strongly manifested. The longitudinal variations of the fluid temperatures for $Pe = 5$, 20 are presented in Fig. 6. The cooling mechanism affects the temperatures in the negative part of the tube due to the presence of axial conduction. It is observed that wall temperatures deviate considerably from center temperatures as Pe diminishes. The influence of axial conduction is very significant near $G_z = 0$ where the heat exchange zone between the fluid and the surrounding begins. Low fluid velocities in the vicinity of the wall decreases the axial convection in the flow direction. This, of course, induces the axial conduction mechanism opposite to the flow direction. On the other hand, in the neighborhood of the tube center, higher fluid

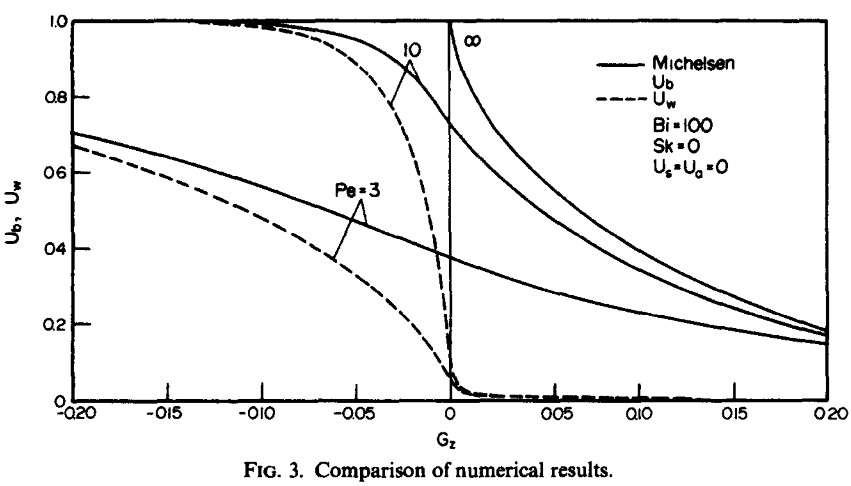

FIG. 3. Comparison of numerical results.

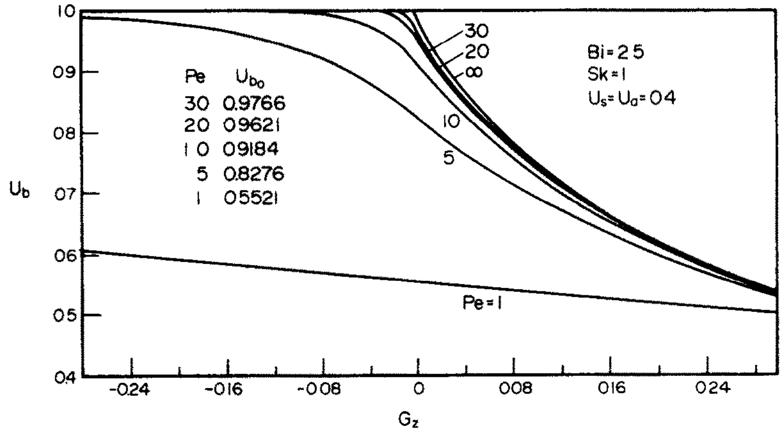

FIG. 4. Bulk temperatures in the upstream and downstream regions.

velocities increase the axial convection downstream. Hence, the upstream heat conduction is dominated by the downstream heat convection. As a result, large and small temperature changes are observed close to $r' = 1$ and $r' = 0$ respectively. Consequently, the overall effect is expressed in terms of U_b which experience a decrease near the origin as Pe decreases. Figure 7 demonstrates that axial conduction tends to increase the length of the thermal entrance region. Of course, this is based on the assumption that cooling conditions remain unchanged. Bulk temperatures are strongly affected by Pe and approach asymptotic values in the thermally developed region. Local heat-transfer rates calculated from equation (20) are illustrated in Fig. 8. Total heat fluxes decrease sharply for small Pe leading to considerable errors when longitudinal conduction is omitted. Nusselt numbers associated to the same thermal boundary conditions are presented in Fig. 9. The temperature surface of Fig. 10 allows the visualization of the overall pre-cooling caused by low Péclet number fluids.

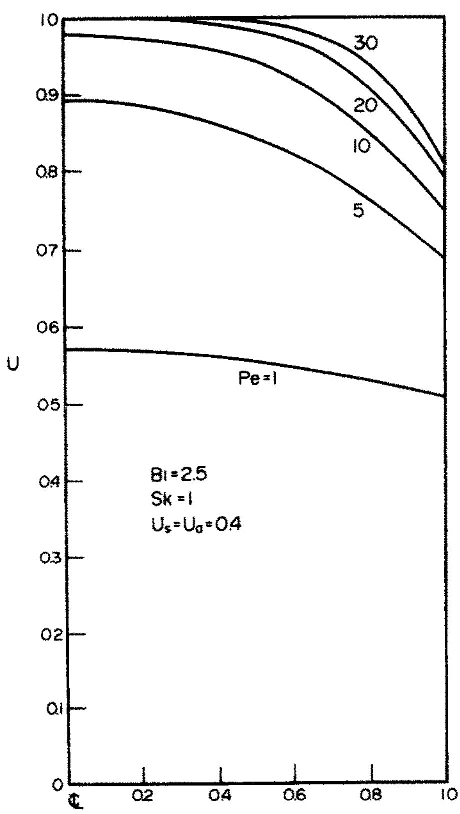

FIG. 5. Variation of the temperature profile with Pe at $G_z = 0$.

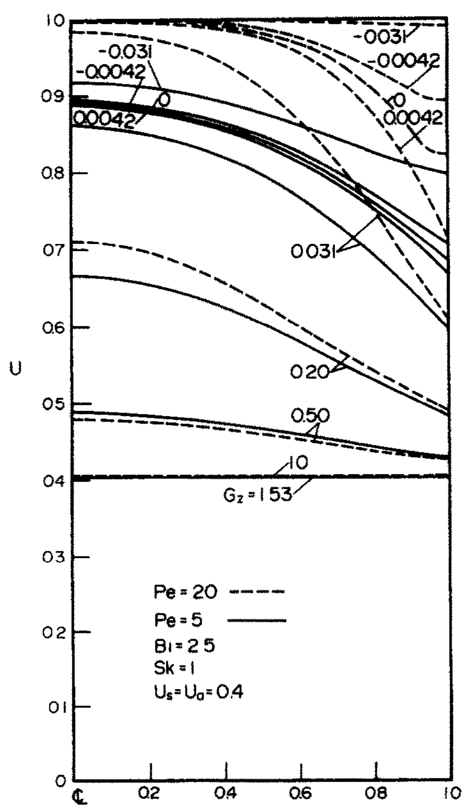

FIG. 6. Effect of axial conduction on the temperature distribution.

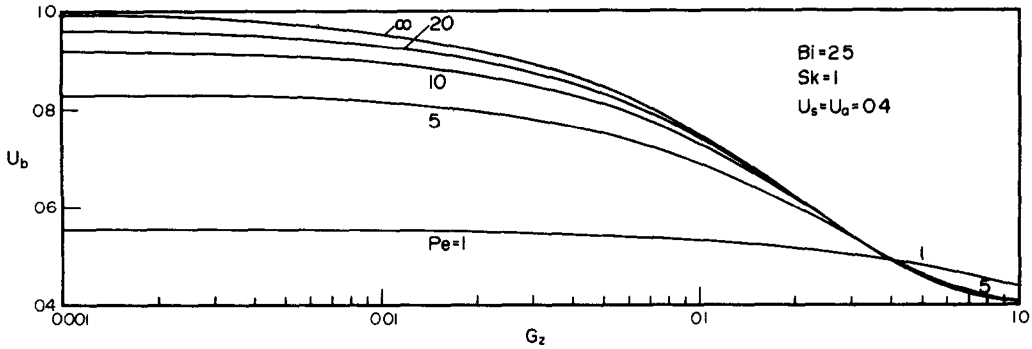

FIG. 7. Bulk temperatures in the downstream region.

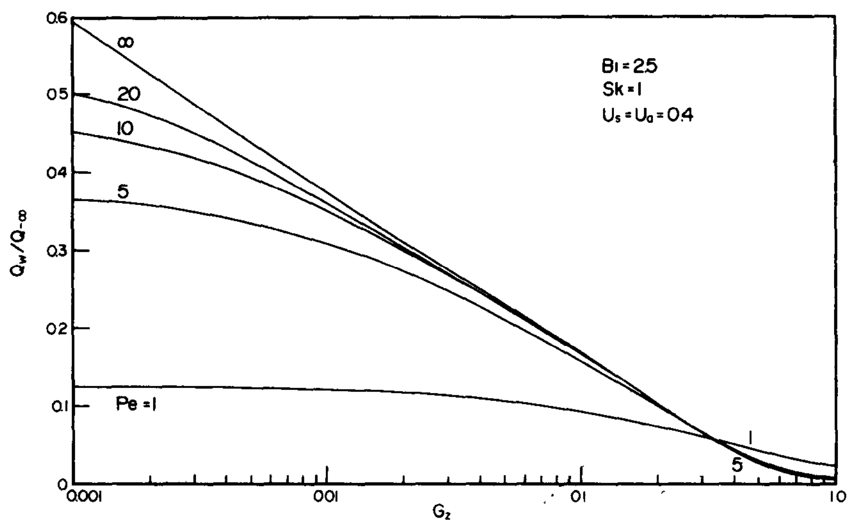

FIG. 8. Heat fluxes in the downstream region.

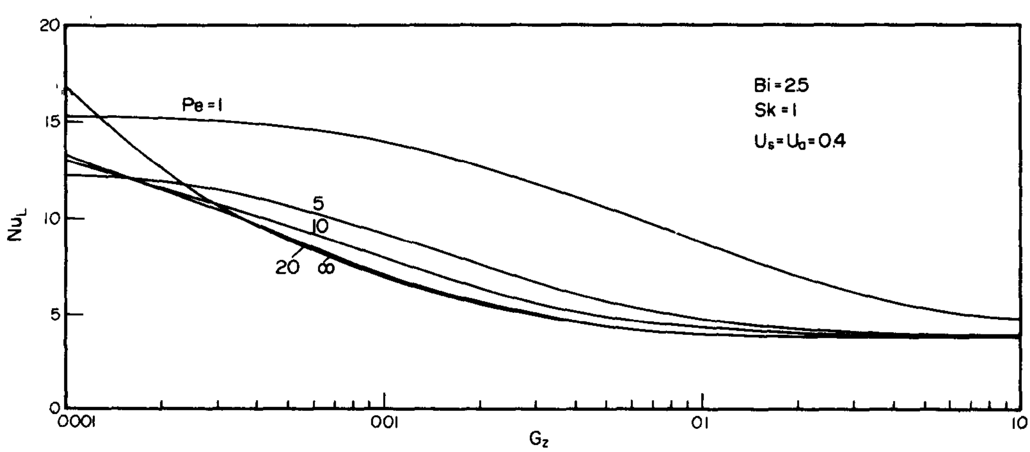

FIG. 9. Nusselt numbers in the downstream region.

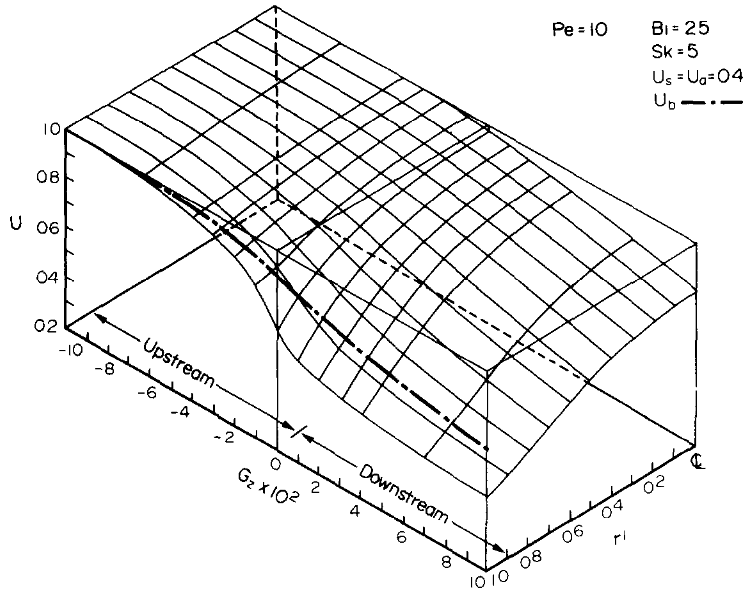

FIG. 10. Temperature surface.

It was stated previously that the upstream penetration depends on both the axial conduction and the cooling level at $G_z > 0$. Therefore, for fixed values of Pe, the penetration distance should decrease as the cooling level decreases too. Accordingly, temperature profiles at $G_z = 0$ become more uniform. This tendency is shown in Fig. 11 for the case of radiation cooling only. Figure 12 shows also the global effect of radiation cooling for the upstream and downstream portions of the tube. As Sk decreases, the pre-cooling of the negative region decreases too.

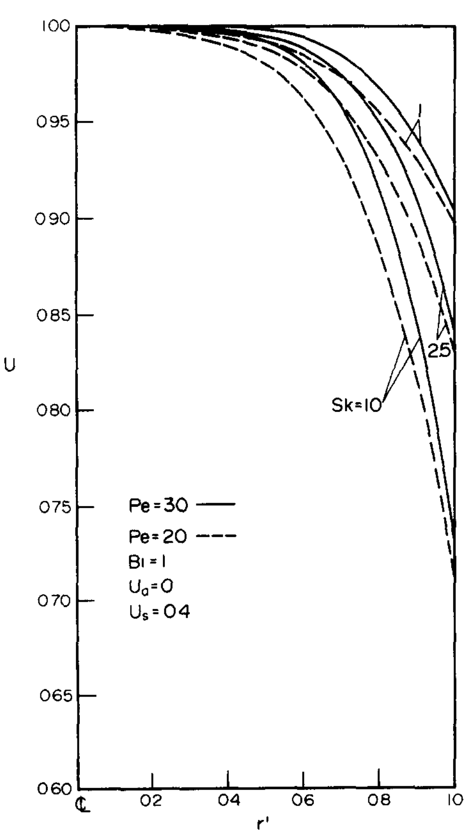

FIG. 11. Variation of the temperature profile with Sk at $G_z = 0$

The role of axial heat conduction for the case of uniform wall temperature is noteworthy when $Pe < 50$[1, 11]. This single number establishes the border line for the contribution of axial conduction in entry-region heat-transfer problems. However; for general boundary conditions, as in the case of simultaneous convection and radiation, a unique number is not enough to show the importance of longitudinal conduction. These situations require the combined presentation of the parameters responsible for axial conduction and heat loss at the walls. Accordingly, it could be interesting to calculate the critical Péclet numbers as functions of the various cooling conditions expressed by Biot and/or Stark numbers. This can be done by drawing the bulk temperatures at $G_z = 0$ and comparing these values to those where axial conduction is omitted. For the case of radiation only, Fig. 13 presents the variation of U_{b_0} with Sk having Pe as a parameter. Here, it is observed that $Pe_c = 20$ when $Sk = 3$ and $Pe_c = 30$ when $Sk = 8$ respectively. The error criterion utilized corresponds to 3% used also in [1], although different criteria can be employed. Likewise, for the case of convection only, Fig. 14 depicts the relationship between U_{b_0} and Bi. Using the same error criterion $Pe_c = 30$ corresponds to $Bi = 6$. Finally, the coupled effect of convection and radiation is illustrated in Fig. 15 for two situations, $Pe = 10$ and 30. These three figures are of considerable importance because they demonstrate that Pe_c increases as the heat dissipation increases. Moreover, $Pe_c \rightarrow 50$ for the limiting condition of constant wall temperature. This value was calculated by Michelsen and Villadsen [11].

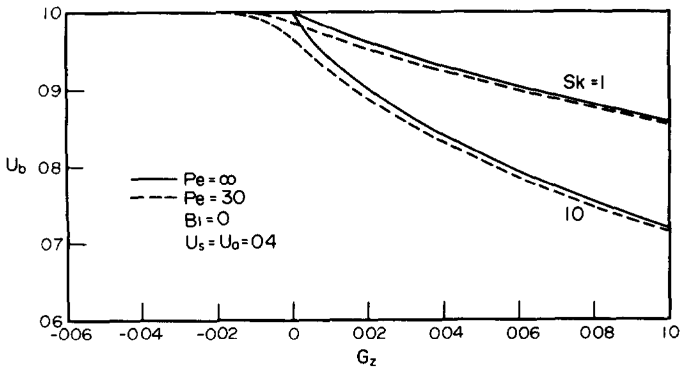

FIG. 12. Axial dependence of bulk temperatures with Sk.

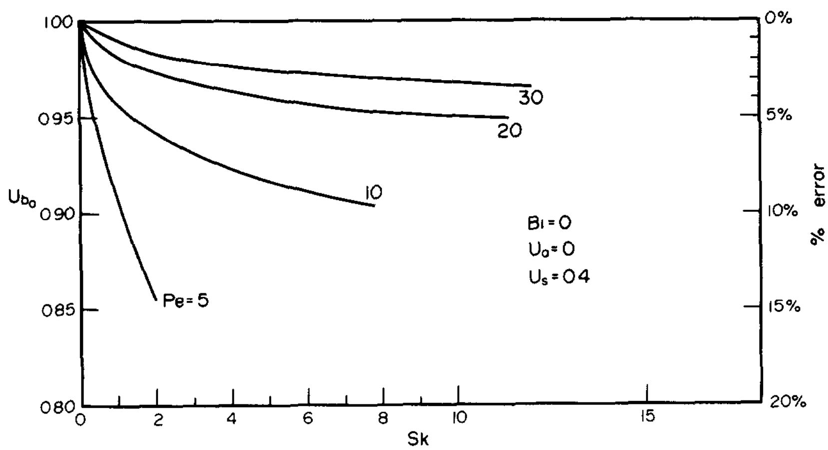

FIG. 13. Bulk temperatures at $G_z = 0$ depending on Sk.

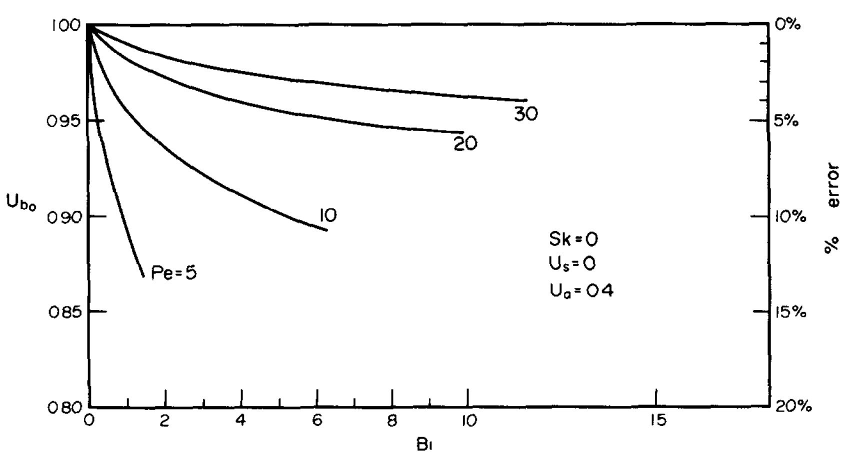

FIG. 14. Bulk temperatures at $G_z = 0$ depending on Bi.

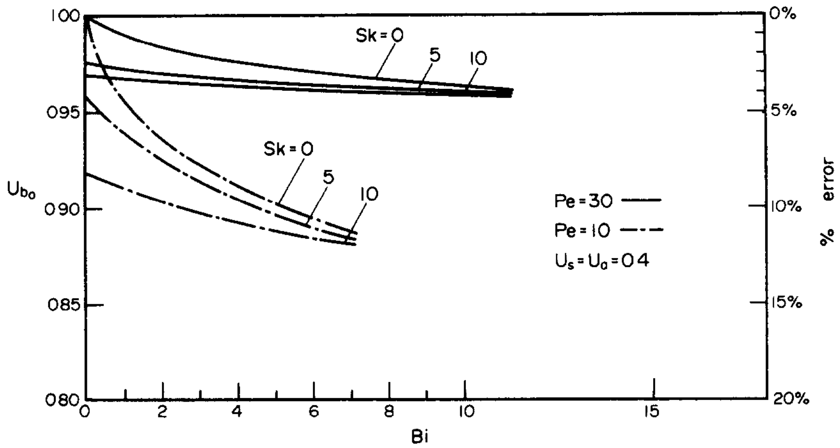

FIG. 15. Bulk temperatures at $G_z = 0$ depending on Sk and Bi.

CONCLUSIONS

A numerical scheme has been developed for calculating the entry-region heat transfer in axially conducting fluids through circular tubes. The interaction of axial conduction and the parallel cooling by convection and radiation at the walls was fully investigated. Convergence of the numerical solution is assured by comparing the results of the present problem with those considering limiting cases.

The different thermal effects of axial conduction on the convection phenomenon depends not only on the magnitude of *Pe* but also on the magnitude of *Bi* and/or *Sk*. Thus, the wall heat flux decreases along the axial distance from the origin as the influence of axial conduction increases. Moreover, the role of axial conduction on the bulk temperature is stronger when the heat liberation is increased. It must be emphasized that the contribution of axial conduction can be neglected even when $Pe < 5$ depending on the cooling intensity between the fluid and the environment. Therefore, for these situations the hypothesis of uniform temperature at the origin could be valid and the mathematical model of the problem greatly simplified.

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CONDUCTION AXIALE DANS LES ECOULEMENTS LAMINAIRES AVEC DES FLUX THERMIQUES NON LINEAIRES SUR LA PAROI DU TUBE

Résumé—On développe une analyse numérique pour déterminer les paramètres de transfert thermique d'un écoulement de fluide rejetant vers l'extérieur de la chaleur par conduction et par rayonnement. L'influence de la conduction axiale est incluse et le profil de vitesse est pris non uniforme dans la direction transversale. L'utilisation d'une transformation élimine les conditions aux limites requises à l'infini. Des techniques numériques approchées sont exploitées pour résoudre le problème non linéaire conjugué. Lorsque le nombre de Péclet augmente, le champ de température se réduit à celui relatif à la conduction axiale nulle. Les résultats du calcul montrent que les effets de la conduction axiale sont fortement altérés par les paramîtres traduisant la convection et le rayonnement. Les températures du fluide, les flux thermiques pariétaux et les nombres de Nusselt sont donnés graphiquement en fonction du nombre de Graetz. On présente les nombres de Péclet critiques pour une variété de conditions de refroidissement en prenant comme référence la température moyenne de mélange du fluide.

AXIALE WÄRMELEITUNG IN LAMINAREREN ROHRSTRÖMUNGEN MIT NICHTLINEAREN WANDWÄRMESTRÖMEN

Zusammenfassung—Für eine Flüssigkeitsströmung, die an das umgebende Medium durch Konvektion und Strahlung Wärme abgibt, ist eine numerische Berechnungsmethode entwickelt worden, um die Parameter der Wärmeübertragung zu bestimmen. Der Einfluß der axialen Wärmeleitung und des in radialer Richtung ungleichförmigen Geschwindigkeitsprofils wird dabei berücksichtigt. Die Anwendung einer Transformation eliminiert die sonst erforderliche Berücksichtigung von unendlichen Randbedingungen. Numerische Näherungsmethoden wurden zur Lösung des nichtlinearen konjugierten Problems angewandt. Bei Zunahme der Péclet-Zahl vereinfachen sich die Temperaturfelder auf Formen, bei welchen axiale Leitung unberücksichtigt bleibt. Die errechneten Ergebnisse zeigen, daß die Einflüsse der axialen Wärmeleitung stark von den Parametern abhängen, die für Konvektion und Strahlung verantwortlich sind. Mittlere Fluidtemperaturen, Wandwärmeströme und Nusselt-Zahlen werden in Abhängigkeit von Graetz-Zahlen aufgetragen. Kritische Péclet-Zahlen mit der mittleren Fluidtemperatur als Bezugsgröße werden für verschiedene Kühlbedingungen angegeben.

ОСЕВАЯ ТЕПЛОПРОВОДНОСТЬ ЛАМИНАРНЫХ ПОТОКОВ В ТРУБАХ ПРИ НЕЛИНЕЙНЫХ ГРАНИЧНЫХ УСЛОВИЯХ НА СТЕНКАХ

Аннотация — Разработан численный метод для определения теплоотдачи от потока жидкости в окружающую среду конвекцией и излучением. Рассматривается влияние осевой теплопроводности при неоднородном профиле скорости в поперечном сечении трубы. Использование преобразования снимает требование задания граничных условий на бесконечности. Нелинейная сопряженная задача решалась с помощью приближенных численных методов. По мере увеличения числа Пекле температурные поля трансформируются в температурные поля без влияния осевой теплопроводности. Полученные результаты показывают, что влияние осевой теплопроводности сильно зависит от параметров, характеризующих конвекцию и излучение. Приводится зависимость объёмных температур жидкости, пристенных тепловых потоков и чисел Нуссельта от чисел Грэтца. Критические числа Нуссельта для целого ряда условий охлаждения даются по объёмной температуре жидкости.